Nikhil’s Masterpiece Journal

Quadratic

Cubic

Cube Root

Square Root

Absolute Value

Linear Equation

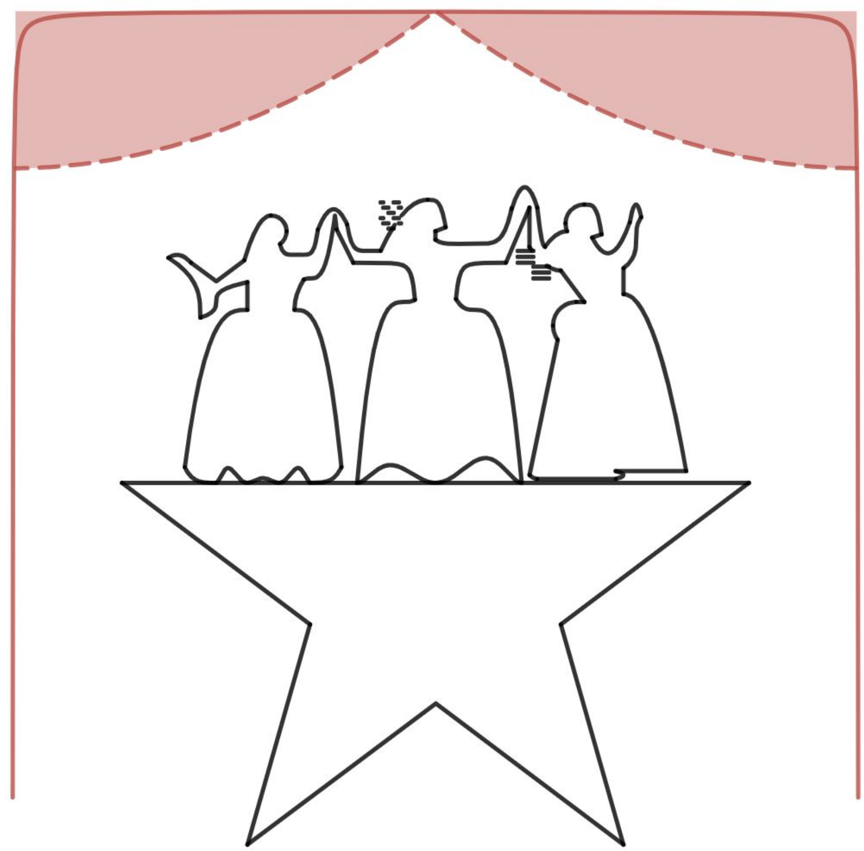
Reciprocal Function

Floor Function

Root Based Function

Transformed Monomial

Circles



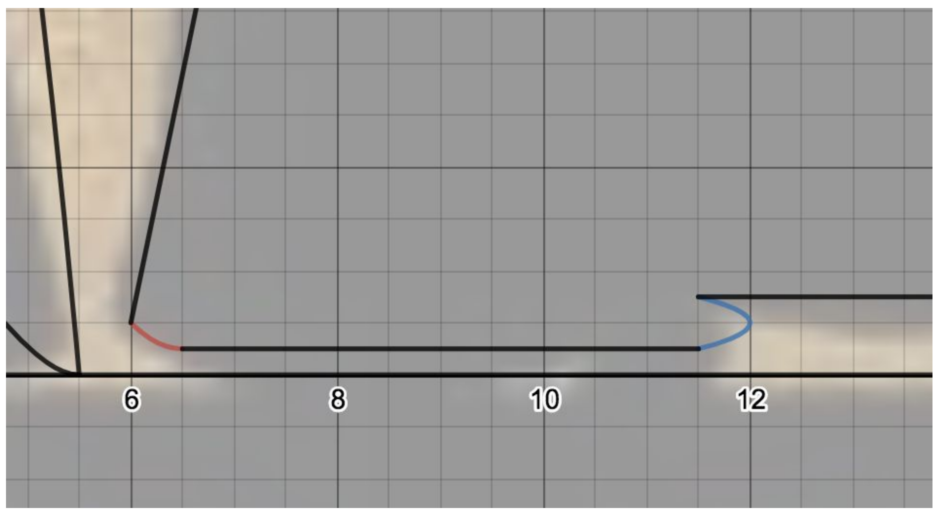
Quadratic

Parent: $f(x) = x^2$

The equation of a quadratic is $y = a(x - h) + k$, where (h, k) is the vertex of the parabola, and "a" is the vertical stretch factor. For almost all of the quadratics, I knew the vertex, and a point on the parabola, and would plug in the point to find the vertical stretch factor.

Bottom of the right dress

Since I used the floor function to create part of the bottom of the dress it looked very structured and still. So in order to create a sense of flowiness, I used quadratics to add some curves and “texture” to the bottom of the dress. I added two curves; one for in-between the floor function and one at the start.



$y = (x - 6.5)^2 + 0.25\{6 \le x \le 6.5\}$

At the start of the floor function, I created another curve to add to the flowiness of the dress. Since it was a curve shape I used a quadratic function to create it. I found the vertex which was the starting point of the floor function, which is (6.5, 0.25). I saw that a point I wanted on the curve was (6, 0.5). I used this to find the vertical stretch factor instead of the horizontal scale factor, as I knew I wanted the curve more vertically than horizontally.

$y = a(x - 6.5)^2 + 0.25$
$0.5 = a(6 - 6.5)^2 + 0.25$
$0.25 = a(-0.5)^2$
$0.25 = 0.25a$
$a = 1$

$x = -8(y-0.5)^2 + 12\{11.5 \le x \le 12\}$

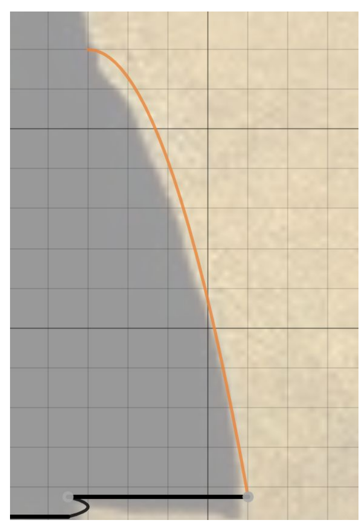
In between the "segments" of the floor function I used an inverse quadratic function to create the curve in between as it created a parabolic shape that a normal quadratic cannot create. To create this function I found the horizontal and vertical translations. In the inverse of a parabola, the equations is $x = (y-k) + h$, where (h,k) is the vertex of the equations. I saw I wanted the vertex at point (12, 0.5), and plugged them into the appropriate points. To find the vertical stretch factor, I plugged in one of the points I wanted on the parabola which is (11.5, 0.75)

$$x = a(y-0.5)^2 + 12$$
$$11.5 = a(0.75-0.5)^2 + 12$$
$$-0.5 = a(0.25)^2$$
$$-0.5 = 0.0625a$$
$$a = \frac{-0.5}{0.0625} = \frac{-50}{6.25} = -8$$

Right Sister's Dress

$y = \frac{-45}{64}(x-12)^2 + 12\{12 \le x \le 16\}$

I used a quadratic function for this as it's a simple curve that goes from the frill towards the waist. I made the vertex the waist, as it would create the correct orientation for what the curve should be. Furthermore, I knew the quadratic would open down, so all I had to do was find the vertical stretch factor. To do so I plugged in the point where the frill ends into the equation which is $(16,\ 0.75)$.

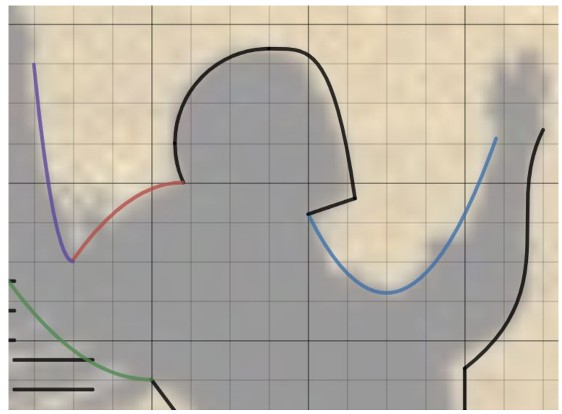


$$y = -a(x-12)^2 + 12$$
$$0.75 = -a(16-12)^2 + 12$$
$$-11.25 = -a(4)^2$$
$$-11.25 = -16a$$
$$a = \frac{-11.25}{-16} = \frac{1125}{1600} = \frac{45}{64}$$

Right Sister's Shoulders/arms

$y = (x - 11)^2 + 14.6\{10 \le x \le 12.4\}$

I used a quadratic for this equation, as the inner arm that goes between the neck and wrist creates a parabola.



$y = \frac{-1}{2}(x - 8.4)^2 + 16\{7 \le x \le 8.4\}$
$y = 10(x - 7)^2 + 15\{6.5 \le x \le 7\}$

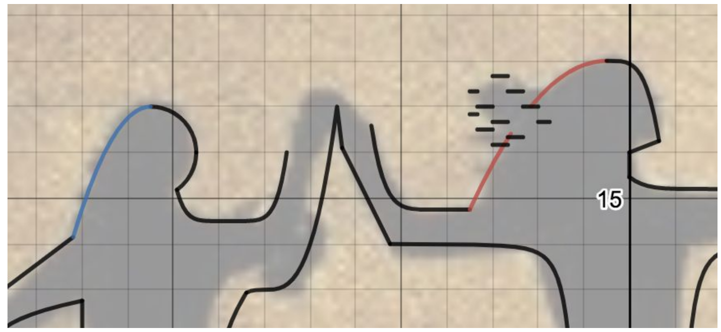
I used a quadratic for these equations, as they are all curves, that are shown as half of the parabola, when the vertex is the center. I used the same method to find the vertical stretch factor in all four of these equations as I did in others.

For example, in the blue function, the arm opened up, so I made the vertex at the lowest point on it which was (11, 14.6). Since, I knew this was the vertex, and another point which was $(10,\ 15.6)$, I plugged it into the standard parabola equation (at the top of the section) to find the vertical stretch factor.

$y = \frac{3}{8}(x - 8)^2 + 13.5\{6 \le x \le 6.14, 7.1 \le x \le 8\}$

I used the same methodology and reasoning for this part of the sister except for when restricting the domain. I wanted the hair to come through and look a part of the sister, and so I restricted the function so that it does not exist during the x-value where the hair is coming out.

Hair/Head of the sisters

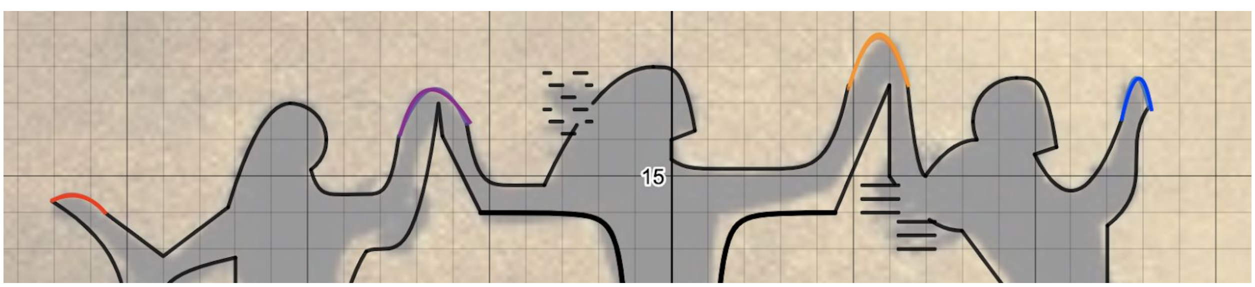


$$y = \frac{-285}{289}(x + 10.5)^2 + 17\{12.2 \le x \le 10.5\}$$

$$y = \frac{-361}{1000}(x + 0.5)^2 + 18\{-3.5 \le x \le -2.6, -2.16 \le x \le -0.5\}$$

I used a quadratic for the part going from the shoulder to the top of the head, because it created a curve-like shape. In both functions the curve opened downward, and therefore I found the vertex, by it being the highest point on head. Since I knew the vertex, I found a point I wanted for each curve, and used that to find the vertical scale factor. For the red equation, I restricted the domain so that the function is hidden in the middle, so the hair looks natural and like it is a part of the sister.

Hands

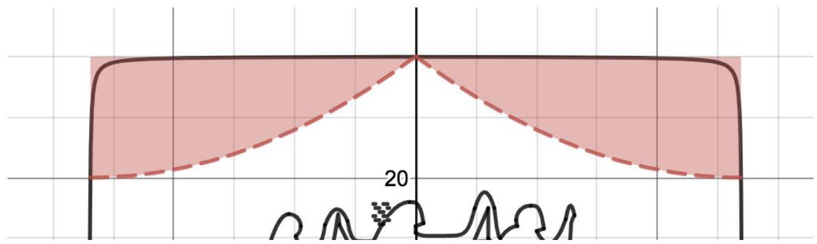


$$y = \frac{-50}{81}(x + 16.5)^2 + 14.5\{-17.05 \le x \le -15.6\}$$
$$y = -1.3(x + 6.5)^2 + 17.4\{-7.4 \le x \le -5.635\}$$
$$y = \frac{-65}{32}(x - 5.7)^2 + 18.8\{4.872 \le x \le 6.5\}$$
$$y = \frac{-57}{8}(x - 12.8)^2 + 17.7\{12.4 \le x \le 13.137\}$$

To create the hands, I used a quadratic as they created a parabolic shape, as it curves outwards both ways from the vertex. The hands all open downward, so I was able to find the vertex as it was the highest point on the hand. Since I knew the vertex, I plugged in another point I wanted the parabola to fall on, to find the vertical stretch factor, like I have done for all the other quadratics.

Curtain

$y > \frac{1000}{72361}(|x| - 26.9)^2 + c\{-26.9 \leq x \leq 26.9\}\{y \leq 30\}$, where $-25 \leq c \leq 20$.



I used a quadratic function to create the curtain of the theatre, as it was a curve that looked like one side of a parabola to the vertex. The curtain opened up, and so I made the vertex the a point on the frame of the theatre which was $(26.9, 20)$. Then I plugged in a point I wanted the curve to go through to find the vertical stretch factor, like I did in many other equations.

I used the transformation $f(|x|)$, as the curtain is on both sides of the theatre, and since the the original curtain constructed would've stopped when $x = 0$, I used that transformation to construct a symmetrical curtain on the left side, when $x = 0$ is the axis of symmetry.

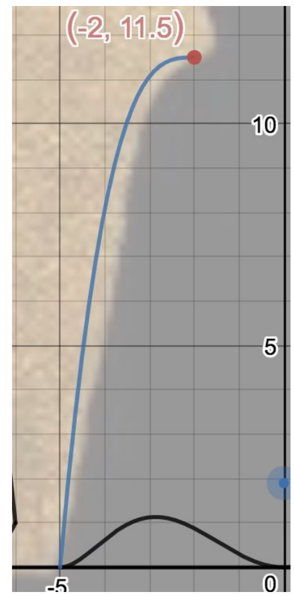
The reason I used c in my equation instead of 20, is because c is a slider that changes the y-value of the vertex. This is to give the effect of the curtain opening up on stage, like done in a show, as the curtain moves up as c gets closer to 20. I put $(26.9, 20)$ to find the vertical stretch factor, only as it was the highest point I constructed the curtain to go, and when the curtain stops moving, so I based it off of that.

Lastly, I used an inequality instead of an equal sign so that the curtain is shaded, and gives the effect of being in a real theatre. Since the curtain was an inequality, I had to restrict the range and the domain, to keep the shading from not going too high or too much to the side. If I only restricted the domain, the shading would've gone until $y = \infty$.

Cubic

Parent: $f(x) = x^3$

Dress



I used the same method to find the equations of the middle and left sisters' dresses.

Middle Sister's Dress

Left Side

$y = \frac{23}{54}(x+2)^3 + 11.5\{-5 \leq x \leq -2\}$

For the dress, I used the cubic function as the parent as the dress changes from being a curve to being flat, as that was what dresses were like in those years. I wanted to use the cubic function, because when at (0,0) the curves become flat. To create this function, I was easily able to identify the horizontal and vertical translations that took place as I could identify a point at which the dress went flat. Knowing the translations, all I had to do was to find the vertical scale factor, as the curve on the dress were a "stretched" version of $y = x^3$. I knew another point on the curve of the dress as that was the point where the frill of the dress ended, and the dress started. This methodology and process were used to make both sides of all 3 of the dresses, as I was able to identify the point where it went flat for all of them, and I knew the points in which the frill ended. I was able to easily identify the domain as I wanted the function to exist between the end of the frill and the point where the line goes flat.

To find the vertical scale factor of the function, I plugged in the translations of the point (0,0), as I knew where the dress was going to be flat. In this case, the point in which it went flat was (-2,11.5) and the frill ended at (-5,0)

$y = a(x+2)^3 + 11.5$
$0 = a(-5+2)^3 + 11.5$
$-11.5 = a(-3)^3$
$-11.5 = -27a$
$a = \frac{-11.5}{-27} = \frac{115}{270} = \frac{23}{54}$

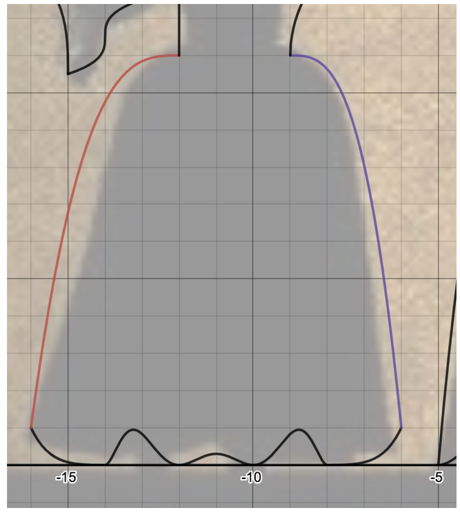
Right side

$y = \frac{11}{27}(x - 2.5)^3 + 11\{1.3107 \leq x \leq 5.5\}$

Left Sister's Dress

$y = \frac{5}{32}(x + 12)^3 + 11\{-16 \leq x \leq -12\}$

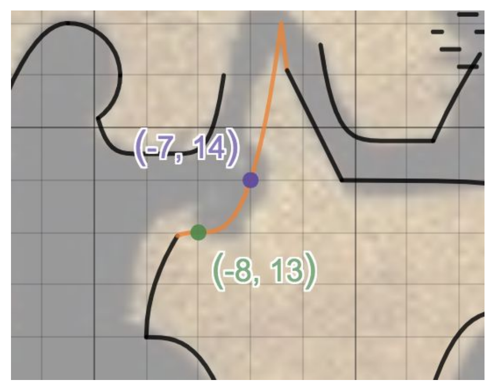
$y = \frac{-10}{27}(x + 9)^3 + 11\{-9 \leq x \leq -6\}$



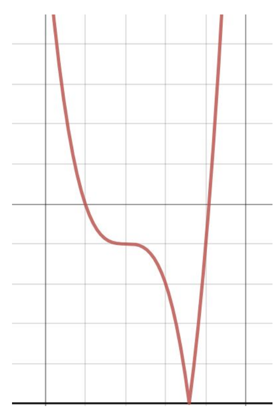
Arm

Left sister's arm

$y = ||(x + 8)^3 - 4| - 6| + 11\{-8.4 \leq x \leq -6.3\}$



I used a cubic function for this part of the arm, as it curves, then goes horizontally straight, and then curves again. However, in this same function, I wanted to create the tip of where the hands meet, and when you do the transformation $|f(x)|$ you create those steep turns. I had to do the $|f(x)|$ transformation twice, as I needed to get the function into the correct orientation after getting the shape correct. For example, the $|(x + 8)^3 - 4|$ was to get the shape of the function as it created the steep turn, however the tip was below the curve. The vertical translation of 6 units downwards, and the transformation of $|f(x)|$, was to get the steep tip above the curve. Lastly, the vertical translation of 11 units upward was to get the center to the correct point, as the horizontal translation of 8 units to the left was done in the first step. I restricted the domain as such, as I -8.4, was the x-value where the functions of the waist and arm interesected, and -6.3 is the x-coordinate I wanted the arm of the middle sister to start on.



I had the center of the tip at (-8,13), so I plugged in the point (-7,14) into the equation to find the vertical stretch factor, as I wanted it on the curve going towards the tip.

$$y = a||(x+8)^3 - 4| - 6| + 11$$
$$14 = a||(-7+8)^3 - 4| - 6| + 11$$
$$3 = a||(1)^3 - 4| - 6|$$
$$3 = a||-3| - 6|$$
$$3 = a|3 - 6|$$
$$3 = a|-3|$$
$$3 = 3a$$
$$a = 1$$

Cube Root

Parent: $f(x) = \sqrt[3]{x}$

Sleeve

Sleeve of left sister's dress

$y = \sqrt[3]{x + 14} + 11.5\{-12 \leq x \leq -15\}$
$y = -3\sqrt[3]{x + 15} + 10.5\{-17.05 \leq x \leq -15\}$

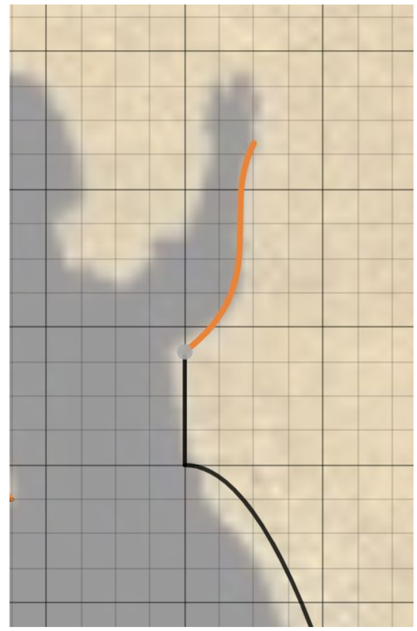
I wanted to create the sleeve of the dress and noticed how it dangled. The sleeve with the dangling aspect of it created a "shape" in which it curves, goes straight up vertically, and then curves in the other direction, where both the curves are more horizontal. This led me to realize that the parent of this function would be $y = \sqrt[3]{x}$. I used this as the parent to create both sides of the sleeve as they both had that aspect to it, the "blue" segment was more clearly shown than the "red" segment. To create this transformation, I needed to identify the translation which I was able to do, as I could see where the "shape" the sleeve made went straight, and where the center point was.

To create the function, I first found the translations and then found the vertical stretch factor by plugging in a point I knew was going to be on either one of the curves in the function. In terms of the blue part of the sleeve, I could make that a plausible "center" could be (-14,11.5) as that was around the center of the vertical part of the blue part. From there I plugged in point (-13,12.5) as that seemed an appropriate point to put on the curve going in the positive direction.

$y = a\sqrt[3]{x + 14} + 11.5$
$12.5 = a\sqrt[3]{-13 + 14} + 11.5$
$1 = a\sqrt[3]{1}$
$1 = a(1)$
$a = 1$

Rights sister's arm

$y = 2\sqrt[3]{x - 12.8} + 15.5\{12 \leq x \leq 13.137\}$



The cube root function created that shape where there is a vertical line between that branches off into two curves, rather than the cubic which creates a horizontal line. The arm of the sister created the exact shape a cube root function makes, as it bends at the elbow and the wrist, but is vertically straight at the arm. To find the transformation in the line, I found a plausible midpoint of the vertical part of the arm, which was (12.8, 15.5). It had to be a midpoint as it expanded both up and down. Once I had the transformations, I plugged in a point I knew I wanted on the arm which was $(13, 16\frac{2}{3})$, and used this to find the vertical stretch factor.

$$y = a\sqrt[3]{x - 12.8} + 15.5$$
$$16\tfrac{2}{3} = a\sqrt[3]{13 - 12.8} + 15.5$$
$$1\tfrac{1}{6} = a\sqrt[3]{0.2}$$
$$1.166667 = 0.58480354764a$$
$$a = \frac{1.166667}{0.58480354764} = 2$$

Square Root

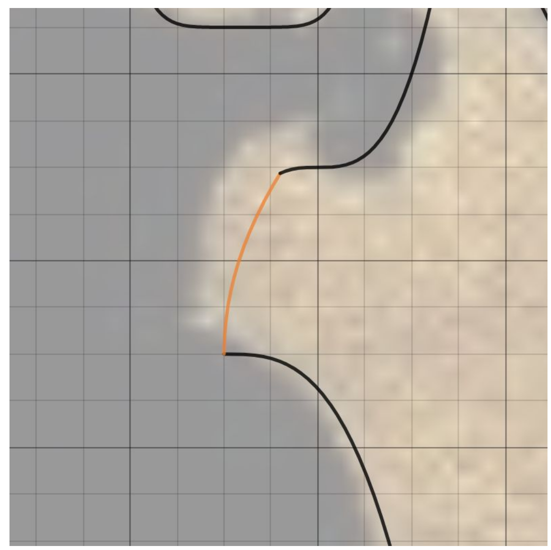
Parent: $f(x) = \sqrt{x}$

Waist

Left sister's wait

$y = \frac{5}{2}\sqrt{x+9} + 11\{-9 \le x \le -8.4\}$

I used the square root function for the waist because it made the shape of the waist going into the arm. With this in mind, I was able to transform this function as I knew the what the starting point was, and I was able to identify that there was a vertical stretch as it looked steeper than the original parent function of $y = \sqrt{x}$.



I restricted the domain to what it is, as those are the x-values where the curve intersected the arm and the hip. I was able to identify the translations, as I found where the center of the function was based on where the dress ended. I picked point (-8,13.5) as a point I wanted on the function, and used that to find the vertical stretch factor.

$y = a\sqrt{x+9} + 11$

$13.5 = a\sqrt{-8+9} + 11$

$2.5 = a\sqrt{1}$

$a = 2.5 = \frac{5}{2}$

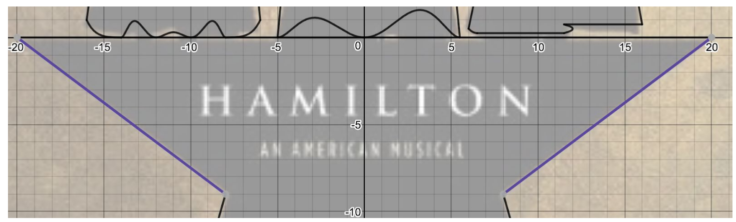
Absolute Value

Parent: $f(x) = |x|$

Star

Topside

$f(x) = \frac{5}{6}|x| - \frac{20}{6}\{-4 \le x \le -1.6,\ 1.6 \le x \le 4\}$

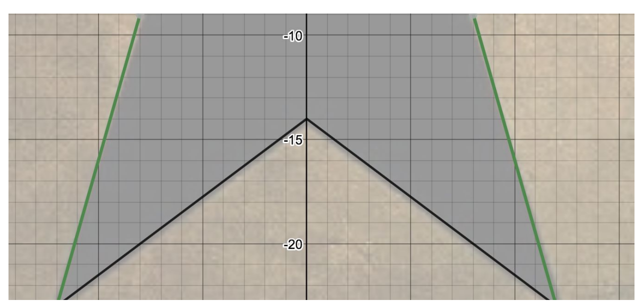


For this part of the star, I knew that I was going to use an absolute value that had a domain restricted in two ways. To create this function I used the rule of when $f(x)$ goes under the transformation $f(|x|)$ the graph to the left of the y-axis becomes symmetrical to the graph on the right side of the y-axis. I used this reasoning, as I found it simpler to work out, due to the fact that the functions didn't have a vertex that was shown. Using this methodology, I constructed a linear line on the right side, and then put the absolute value "brackets' around the x. I was able to find the starting points of the star because I had the equation of a horizontal line of $y = 0\{-20 \le x \le 20\}$ .This worked since a star is symmetrical when vertically cut in half. I used this method for all three parts of the star.

To create the straight line I used point-slope form and turned it into slope-intercept form. I knew that the two points the line went through were $(20, 0)$ *and* $(8, -9)$

Slope: $\frac{0-(-9)}{20-8} = \frac{9}{12} = \frac{3}{4}$

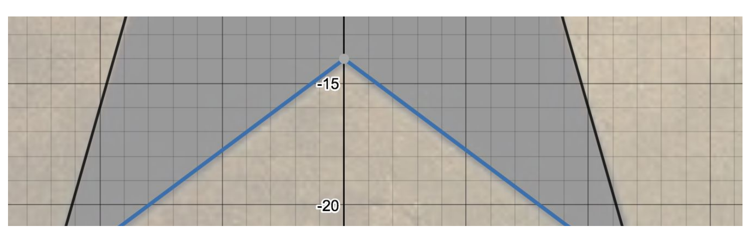
$y - 0 = \frac{3}{4}(x - 20)$ which when simplified is $y = \frac{3}{4}x - 15$, and then with the transformation $f(|x|)$, you get $y = \frac{3}{4}|x| - 15$



Bottom-side

$f(x) = -\frac{7}{2}(|x| - 8) + 9\ \{-12 \le x \le -8,\ 8 \le x \le 12\}$

Bottom

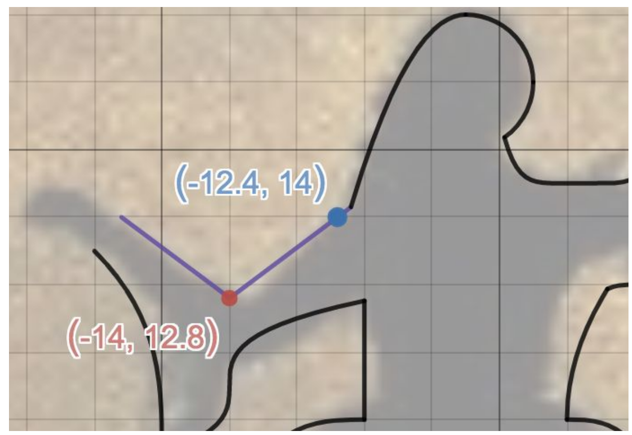


$f(x) = -\frac{3}{4}|x| - 14 \{-12 \le x \le 12\}$

Arm of left sister

$y = \frac{3}{4}|x + 14| + 12.8\{-15.6 \le x \le -12.2\}$

I used an absolute value function for this part, as the bent arm creates a V shape, and an absolute value function also creates a V shape. An equation of an absolute value function is $y = a|x - h| + k$, where (h, k) is the vertex, and "a" is the vertical stretch factor. I could see that the V shape opened up, so I knew that the vertex would be the lowest point on the graph, which was (-14,12.8). Since I knew the vertex, I found another point I wanted to be included in the function which was $(-12.4, 14)$, and plugged that in to find the vertical stretch factor. I restricted the domain to where the figure started to curve, as absolute values do not curve.



$y = a|x + 14| + 12.8$

$14 = a|-12.4 + 14| + 12.8$

$1.2 = a|1.6|$

$1.2 = 1.6a$

$a = \frac{1.2}{1.6} = \frac{12}{16} = \frac{3}{4}$

Linear Equation

Parent: $f(x) = x$

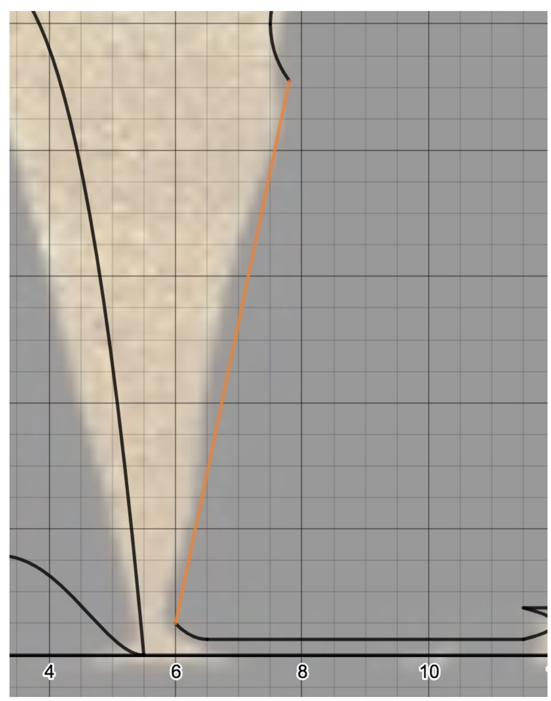
To create all of these functions I used the point-slope form as I couldn't identify a y-intercept right away. Point-slope form is $y - y_1 = m(x - x_1)$, where m is the slope, and (x_1, y_1) is a point on the line. All I needed to do was find m, as I always knew two points on the line.

Since I needed to find the slope, I used the slope formula for all the linear equations, since I knew two points on the line. The slope formula is $\frac{y_2 - y_1}{x_2 - x_1}$.

Right Sister's Dress

$y - 0.5 = \frac{43}{9}(x - 6)\{6 \leq x \leq 7.8\}$

I used a linear equation for this part of the dress instead of a curve because this dress doesn't really curve much until it gets close to the rear end of the sister. Using a line also worked because the bottom of the dress also had straight parts in it, and so it looked as if it belonged.

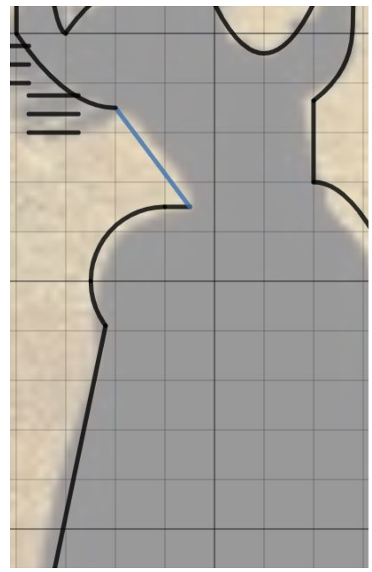


I used the slope formula to find the slope, as I knew I wanted a line to go between points (6, 0.5) and (7.8, 9.1), as that was where the bottom of the dress ended, and where the rear end started, respectively. Once, I found the slope I was able to create the point-slope equation.

$\frac{9.1-0.5}{7.8-6} = \frac{8.6}{1.8} = \frac{86}{18} = \frac{43}{9}$

So the equation would be,
$y - 0.5 = \frac{43}{9}(x - 6)$

Right Sister's Torso

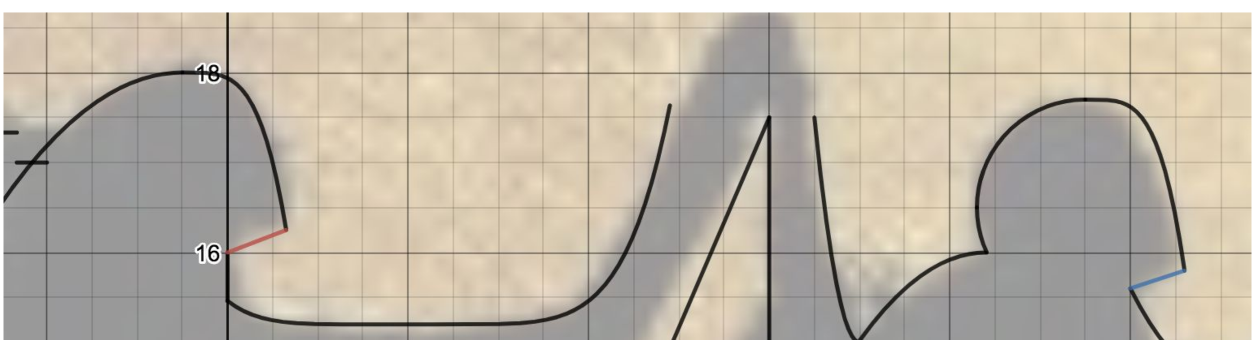


$y - 11.5 = \frac{4}{3}(x - 9.5)\{8 \le x \le 9.5\}$

I used a linear function for this part of the sister, as the backside of her is straight and isn't curving much. Secondly, the right sister had a constant pattern up until hands where the functions would alternate between being linear and having curves. Since, there was just a curve the function now had to be linear.

I used point-slope form to create all of these functions, as I had two points I knew they went through. The points I wanted the function to pass through were $(9.5, 11.5)$ and $(8, 13.5)$. I plugged these into the slope formula and got my final equation.

Chin of the sister's.



$y - 16 = \frac{5}{13}x\{0 \le x \le 0.65\}$

$y - 15.6 = \frac{1}{3}(x - 10)\{10 \le x \le 10.6\}$

I used a linear function for the chin, because it is straight and structured like most chins. Like I did for the rest of the linear equations, I found two points the line would go through and used point-slope form to create the equation.

Outer arms of the middle sister

$y - 14 = -2(x + 5.25)\{-6.3 \le x \le -5.25\}$

$y - 14 = \frac{7}{3}(x - 4.5)\{4.5 \le x \le 6\}$

I used a linear function for the outer arms of this sister, as the arms were straight and rigid, instead of curving and having an authentic flowiness. I used the same method for these equations as I did for other linear functions, as I knew two points the lines would pass through, and used those to find a slope in order to write the equation in point-slope form.

Reciprocal Function

Parent: $f(x) = \frac{1}{x}$

Upper Body

Middle Sister's waist and arms.

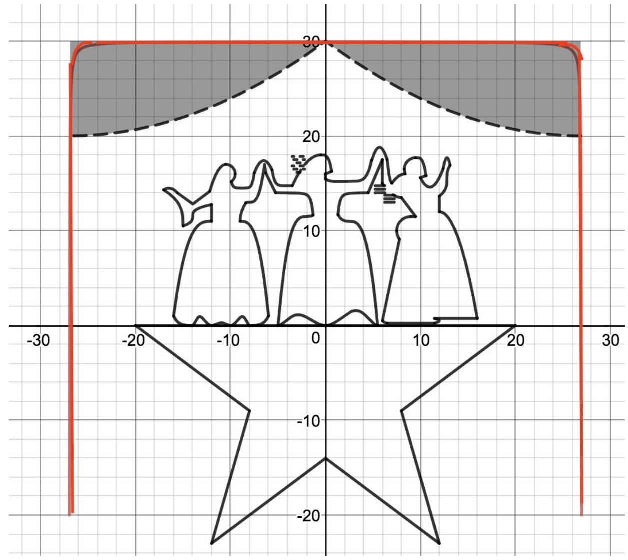
$$y = -\left(\frac{1}{(-|x|+0.5)}\right)^4 + 14\{-5.25 \le x \le -1.3069, 1.3107 \le x \le 4.5\}$$

I used the reciprocal function for the upper body of the middle sister, as it created an asymptotic shape. I reflected the asymptotes over the y-axis, as I wanted the x-value to go towards infinity, as the it approached the horizontal asymptote, and for the y-values to go towards negative infinity as it approached the vertical asymptote. Since the middle sister is centered on x=0, I used an f(|x|) transformation on the asymptote to create a symmetrical asymptote, when x=0 is the axis of symmetry, to create the upper body on the left side of the sister.

I wanted the asymptotes to not be too curved at the center point, so I raised it to the fourth power, as when you raise something to a higher (even) degree, the flatter it gets. By raising it to the fourth power, I was able to get that flatness it lacked. To find the vertical and horizontal translations I found the horizontal and vertical asymptote I wanted the lines to go flat on. Since, I wanted the horizontal asymptote at $y = 14$, the vertical translation was 14 units up , and since I wanted the vertical asymptote at $x = 0.5$, to horizontal translation was 0.5 units to the right. To find the vertical stretch factor, I plugged in a point I wanted the asymptotes to go through. I restricted the domain as such, as those were the x-values at the points in which the asymptotes intersected with the dress, and the inner arms.

$$y = a\left(\frac{1}{(-|x|+0.5)}\right)^4 + 14$$
$$13 = a\left(\frac{1}{(-|1.5|+0.5)}\right)^4 + 14$$
$$-1 = a\left(\frac{1}{(-1.5+0.5)}\right)^4$$
$$-1 = a\left(\frac{1}{(-1)}\right)^4$$
$$-1 = a$$

Theatre



$y = \frac{1}{|x|-27} + 30\{-20 \leq y \leq 30\}$

To add some creativity into the masterpiece, and to make it my own I decided to construct a theatre. To construct the frame of the theatre I used a reciprocal function to create the asymptotic shape. I then used a $f(|x|)$ transformation, so a symmetrical asymptote would be created on the left side of the y-axis. This creates a frame like shape, as the asymptote went to the side, and then down towards negative infinity. I restricted the range instead of the domain, because since there is a vertical asymptote at $y = -27,\ 27$, to restrict the frame while the y-values are going towards negative infinity, the values would be hard to get exact, as they'll be something like 26.9998599482929, and Desmos only gives you decimals to the thousandths place. To find the vertical stretch factor, I used the same method as I did on the other reciprocal function.

Floor Function

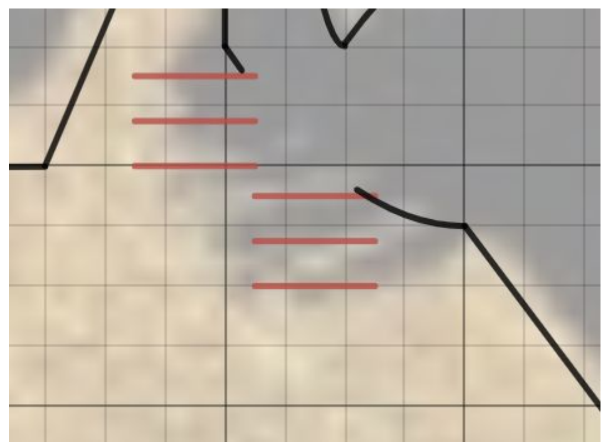
Parent: $f(x) = floor(x)$

Hair

Right sister's hair

$y = floor(-(x - 6.25)) + [14, 14.375, 14.75]\{5.25 \le x \le 7.25\}$

I used the floor function as the hair of the sister, as in the image the hair isn't very curly, wavy and flowy but rather lines that are coming out from the side of it. I felt the floor function was the best method to do this, as I was able to create multiple segments of the hair with one equation.



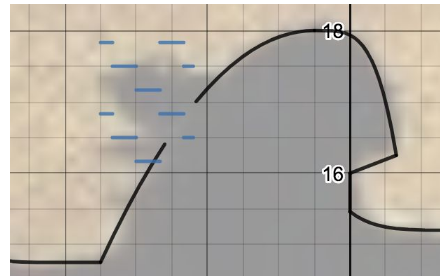
I reflected the function over the y-axis, as the function was going from down to up, when I wanted it to go from up to down, as the hair was higher up on the left side than on the right side. In the function when choosing the vertical and horizontal translations that will take place, I put hard brackets ([,]) over the vertical translations. This is because in Desmos, when you use hard brackets and put commas between values, it graphs all the possible equations. I used this to my advantage when creating the hair, as I put multiple values for the vertical translations, which is how I got the effect of having three lines over each other.

I picked those y-values for the vertical translations, as they were evenly spread out and I didn't want too much clutter. I then had to limit the domain so the step function didn't keep going, as I didn't want the hair to go too long.

When graphing the equation I saw that there weren't any vertical or horizontal dilations necessary, as I wanted each step to be one unit horizontally, and if there was any vertical dilation there would be either a large gap or no gap between the two steps in the hair.

Middle sister's hair

$y = \frac{1}{3}floor(-3(x+2)) + [15.5, 16, 5, 17.5]\{-3.5 \leq x \leq -1.75\}\{16 \leq y \leq 18\}$



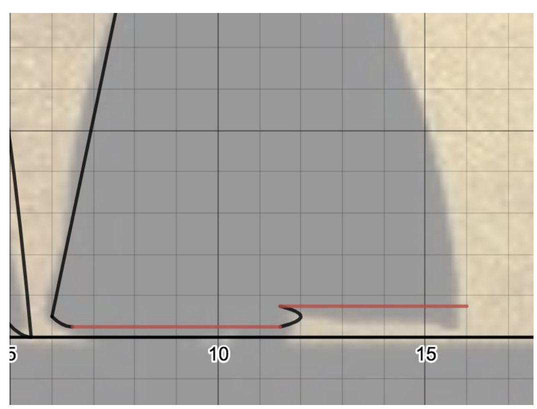
I used the same reasoning and methodology to create this sister's hair. The only difference when creating the equations is that this sister's hair had both vertical and horizontal dilations. To find these dilations I had to figure out how far apart I wanted each "step", and how long I wanted each "step" to be. This sister's hair is very cluttered, and so I wanted the steps close by. I found that I wanted each step to be small, and so I did a horizontal dilation of $\frac{1}{3}$ (It's 3 in the equation as the dilation that takes place is the reciprocal) to make the steps smaller. Since, I had a one-third dilation horizontally, and I wanted to make the distance between the steps smaller, I used a one-third vertical dilation to make it look nice. Lastly, I restricted this function by the range as well, as since the steps are so small, and there are four different vertical translations, if I only limited the domain, the bottom steps in the function would be inside of the body. So I had to restrict the range to prevent that.

Frill of the dress

Frill of Right sister's dress

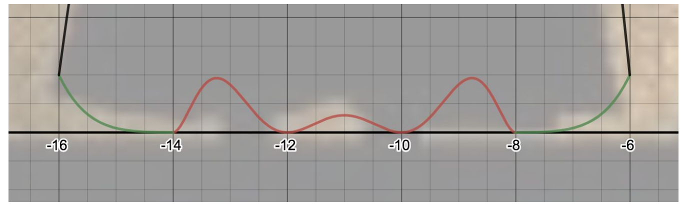
$y = \frac{1}{2}floor(\frac{1}{5}(x-6.5)) + 2.5\{6.5 \leq x \leq 16\}$

The bottom of the right dress was made out of the floor function because it looked like it had two horizontal lines that were one above one another. The floor function creates this shape, and I was able to fill in what was in between these lines with curves to create more of that flowly feeling.



I was able to identify that the segments needed to be 5 times longer, which is how I found the scale factor for the horizontal dilation. I identified this as the segment went from (6.5,0.25) to (11.5, 0.25), which is 5 times longer than the center segment in the parent function. To find the vertical stretch factor, I noticed how the next segment should be 0.5 units above the current, which is how I found the $\frac{1}{2}$ stretch factor, as in the parent function it goes up by 1 unit.

Root Based Function



The frill of the left and middle sisters dresses

Left dress

Middle Section:

$f(x) = \frac{1}{270}(x+14)^2(x+12)^2(x+10)^2(x+8)^2\{-14 \le x \le -8\}$

Outer Section:

$f(x) = \frac{1}{147456}(x+14)^2(x+12)^2(x+10)^2(x+8)^2\{-14 \le x \le -8\}$

I wanted to create the frill of the dresses, so I used a root based function. The frills were meant to go up and down, and so I raised the degree of the roots to an even number, so it bounces back at the zero. I used x^2 as the degree as I didn't want the lines to be distinctly shown to be tangent when it hit the zeros, like in x^4 when you see the flatness. Since the top of the star was on $f(x) = 0$, I was able to easily identify the roots I wanted to use, as the frill had points where $y = 0$. I restricted the domain to those -3, and -1 as those are the largest and smallest roots, respectively, and the function would've gone all the way to infinity. I had to make two functions to create the frill for the sister on the left, one for the inside and one for the outside of it, as the scale factor of the two equations were different, for the I used this method to find the equations of all the frills, both the middle section and the outer section.

In order to find the vertical stretch factor of the function for the middle section, I plugged in the point (-11,0.3), as I wanted that as the top of the middle "frill".

$0.3 = a(-11+14)^2(-11+12)^2(-11+10)^2(-11+8)^2$

$0.3 = a(3)^2(1)^2(-1)^2(-3)^2$

$0.3 = a(9)(1)(1)(9)$

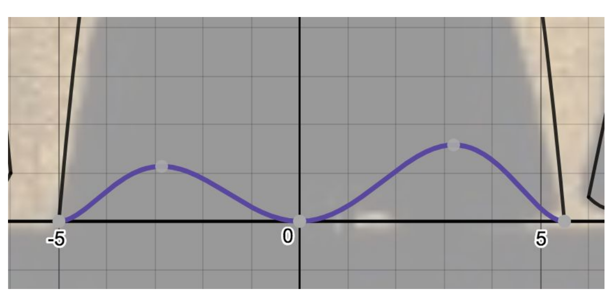
$0.3 = 81$

$\frac{0.3}{0.81} = a$

$a = \frac{3}{810} = \frac{1}{270}$

Frill of the middle dress

$y = \frac{1}{2304}(x)^2(x+5)^2(x-5.5)^2$



Transformed Monomial

Arms

Middle Sister's Arms

$y = (\frac{2}{9}x)^6 + 15.2 \ \{0.5 \leq x \leq 4.872\}$

For the arm I knew that I was going to use a transformed monomial that had a degree larger than four as the arm is flat and then it has an incline. Based on this I knew that I was going to use a degree of x^6 as it stays flat for a while but not too much, but I could anyways change the number of units it is flat for using the horizontal scale factor, which is what I needed to find. I knew the vertex was going to be at (2, 15.2), as the function goes up both ways, and (2,15.2) was in the center between the two curving points, and therefore the vertex.

In order to find the horizontal stretch factor, I plugged in the points (4.5, 16.2) as I wanted that to be a point of the graph of the monomial.

$y = (a(x-2))^6 + 15.2$

$16.2 = (a(4.5-2))^6 + 15.2$

$1 = (2.5a)^6$

$1 = 244.140625a^6$

$\frac{1}{244.140625} = a^6$

$\sqrt[6]{\frac{1}{244.140625}} = a$

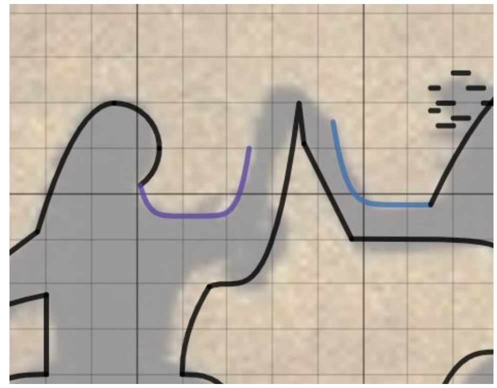
$a = \frac{2}{9}$

I used this same exact method to find the other arm of the middle sister.

$y = (\frac{2}{3}(x+4))^6 + 14.75\{-5.635 \leq x \leq -3.5\}$

Left Sister's right arm

$y = \frac{1}{3}(x + 8.8)^6 + 14.5\{-9.926 \leq x \leq -7.5\}$

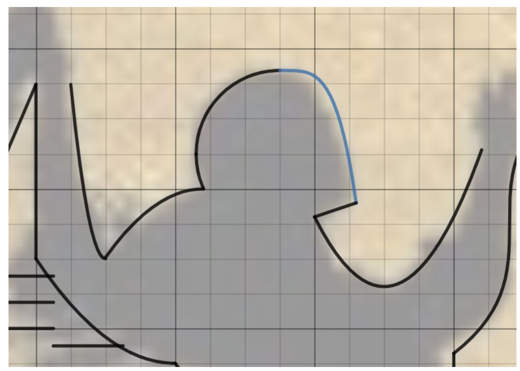


I used the same reasoning and methodology to create this function, except that it had a vertical dilation rather than a horizontal dilation. To do so, when solving algebraically I put "a" (the scale factor I'm solving for), outside of the brackets instead of inside.

Head

Right sister's head

$y = -1.3(x - 9.5)^4 + 17.7\{9.5 \leq x \leq 10.6\}$

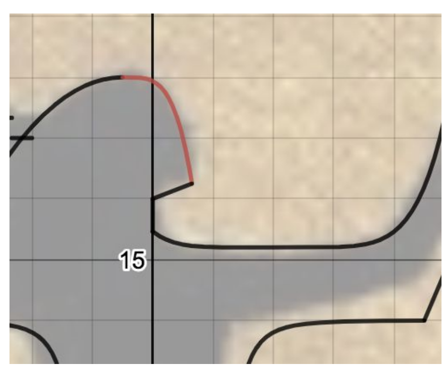


I used a quartic function for this instead of a quadratic, because the head was flatter on the top, and the higher the degree, the flatter it is. I picked the vertex as $(9.5, 17.7)$, as that's the center of the head, and where the flatness starts. Raising the power to the fourth degree, already gave enough flatness to the top of the head, so I only had to find the vertical stretch factor. To do this I plugged in the point $(10.5, 16.4)$ as I wanted it to be on the function.

$$y = a(x - 9.5)^4 + 17.7$$
$$16.4 = a(10.5 - 9.5)^4 + 17.7$$
$$-1.3 = a(1)^4$$
$$-1.3 = a$$

Middle sister's head

$y = -(x + 0.5)^4 + 18\{-0.5 \leq x \leq 0.65\}$



I used the same method to find the equation of this function, as I did for the equation of the Right Sister's head (the equation above).

Circles

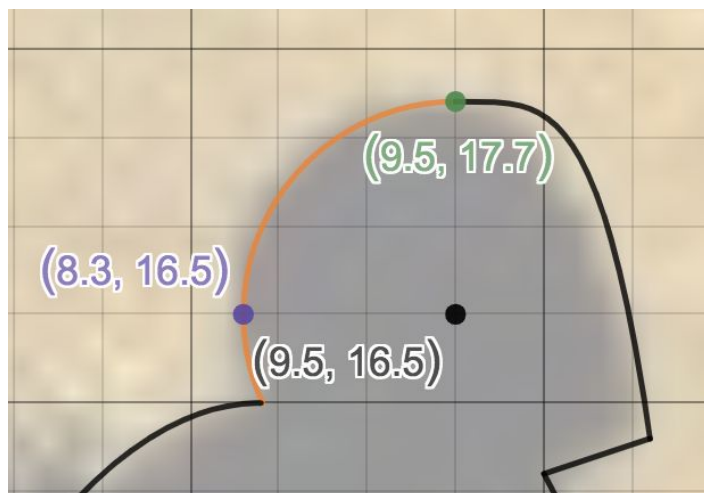
Parent: $1 = x^2 + y^2$

I used the same method to find the equations of all three circles.

Head

Right sister's head

$1.44 = (x-9.5)^2 + (y-16.5)^2 \{8.2 \le x \le 9.5\} \{16 \le y \le 17.8\}$



I used a circle for this part of the head as it starts to curve inwards after curving outwards, which a parabola cannot do. An equation of a circle is $(h-h)^2 + (y-k)^2 = r^2$, where (h, k) is the center of the circle, and r is the radius. So to create this equation I had to find the center of the circle and the radius. I knew I wanted the topmost point to be $(9.5, 17.7)$ as the top of the head. Since I knew that point I was able to find the x-coordinate of the center, as the topmost point is vertically above the center. I found the leftmost point of the circle I was going to construct, as that is the point when the head starts to curve inwards. The point was $(8.3, 16.5)$, and since it was the leftmost point, it had the same y-coordinate as the center. Using these coordinantes, I found the center which was $(9.5, 16.5)$. Since, I had the highest point, I didn't need to use the distance formula to find the radius, as I could see the points were 1.2 units apart. I used this to construct the equation.

$$1.2^2 = (x-9.5)^2 + (y-16.5)^2$$
$$1.44 = (x-9.5)^2 + (y-16.5)^2$$

I restricted the domain and range in this equation as a circle is not a function, so for most x-values there are two y-values, and I did not always want both of those y-values. Therefore, I had to restrict both the domain and range.

Left sister's head

$$1 = (x + 10.5)^2 + (y - 16)^2 \{-10.5 \le x\} \{15.185 \le x\}$$

Right sister's rear end

$$2.25 = (x - 9)^2 + (x - 10)^2 \{7.5 \le x \le 9\} \{9.1 \le x \le 11.5\}$$